$$u = A^{-1/2}(\eta + \zeta)$$

will have values in the space $W_2^{1o}(\Omega)$ for all t. Further differentiability properties of the functions $\mathbf{u}(t, \mathbf{r})$ and $\boldsymbol{\omega}(t)$ were not studied; hence these functions should be considered as generalized solutions of the original problem.

6. The case of a completely filled cavity. In this case $\mathbf{u} = \mathbf{s}$, the equation (5.6) is dropped and the system (5.5) - (5.7) simplifies considerably; namely,

$$\eta = e^{-\nu A_0 t}\eta_0 + \frac{1}{\nu}\int_0^t [I - e^{-\nu A_0 (t-\tau)}] F_1(\omega, \tau)\, d\tau \qquad (6.1)$$

$$\omega + B_1 A^{-1/2}\eta = \omega_0 + B_1 A^{-1/2}\eta_0 - \int_0^t (t - \tau) F_3(\omega, \tau)\, d\tau \qquad (6.2)$$

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ON THERMOELASTIC STABILITY WITH SLIDING FRICTION

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Many works devoted to the investigation of the interdependence between the process of sliding friction and the normal displacement of a friction couple have appeared recently (the basic literature is presented in [1,2]). However, no questions connected with thermoelastic phenomena which can exert essential influence on the friction process in the presence of constraints limiting the normal displacement of the friction couple were considered in these works. The character of the thermoelastic processes occurring with friction is determined by the balance between heat liberation and heat elimination in the friction zone, and depends, in the long run, on the physical-geometric properties of the friction node. In the general case, the effective analytical solution of problems associated with the description of thermoelastic processes with friction, is of considerable difficulty, and it is expedient to examine a model problem with the assumption of approximating some cases of friction with sufficient accuracy and admitting of an exact solution.

The following problem is considered below. A body in the form of a plate moves between two plane, parallel, absolutely rigid surfaces at an invariant distance (belonging to two absolutely solid bodies, say); friction occurs between the first surface and the plate while there is no friction between the second surface and the plate. The first surface is a heat insulator, and the temperature of the second surface (and the corresponding plate surface) is zero. It is required to determine the stress and temperature distribution in the plate. Isothermal and adiabatic conditions are not absolute and can be replaced by other boundary conditions.

The result obtained, the existence of stable and unstable friction modes, is extended to a broad class of friction couples.

Let us select a coordinate system with origin on the surface of the first solid, and the X axis perpendicular to the plate and directed into the plate; let us utilize the notation λ, a^2, α, E, l, v, f, $T(x, t)$, $\sigma(t)$ for the coefficients of heat conduction, thermal diffusivity, linear expansion, the elastic modulus, thickness, rate of plate displacement relative to the first solid, coefficient of friction, temperature, normal stress (acting on an area parallel to the plate). As is customary in the theory of thermal contact with local friction [3-6], the thermophysical parameters are assumed constant.

Let us consider two solutions of the problem corresponding to two different conditions referring to the initial instant

$$T(x, 0) = T_0, \qquad \sigma(0) = 0 \qquad (T_0 = \mathrm{const})$$
$$T(x, 0) = 0, \qquad \sigma(0) = \sigma_0 \qquad (\sigma_0 = \mathrm{const})$$

In the first case we have

$$T(x, t) = T_0 \int_0^l G(x, \xi, t)\, d\xi - a^2 \int_0^t T_x'(0, s)\, G(x, 0, t-s)\, ds \tag{1}$$

Here $G(x, \xi, t)$ is the Green's function of the heat conduction problem under the conditions $T_x'(0, t) = \varphi(t)$, $T(l, t) = 0$.

The Green s function can be represented as

$$G(x, \xi, t) = \frac{1}{4l}\left[\vartheta_3\left(\pi \frac{x-\xi}{4l} \,\Big|\, \pi \frac{a^2 t}{4l^2}\right) + \vartheta_3\left(\pi \frac{x+\xi}{4l} \,\Big|\, \pi \frac{a^2 t}{4l^2}\right) - \right.$$
$$\left. - \vartheta_3\left(\pi \frac{x-\xi-2l}{4l} \,\Big|\, \pi \frac{a^2 t}{4l^2}\right) - \vartheta_3\left(\pi \frac{x+\xi-2l}{4l} \,\Big|\, \pi \frac{a^2 t}{4l^2}\right)\right] \tag{2}$$

where $\vartheta_3(s|t)$ is the third Jacobi theta function [7] (in [8] the Green's function of the considered problem is apparently inaccurate).

By using the relationships

$$\sigma(t) = \frac{\alpha E}{l} \int_0^l T(x, t)\, dx, \qquad T_x'(0, t) = -\frac{fv}{\lambda}\sigma(t)$$

we find from (1) and (2)

$$\Sigma(\tau) = \int_0^\tau k(\tau - s)\, \Sigma(s)\, ds + q(\tau) \tag{3}$$

Here

$$k(\zeta)=\omega\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\exp\left(-\frac{(2n+1)^2\pi^2}{4}\zeta\right)$$

$$q(\eta)=\chi_0\frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\exp\left(-\frac{(2n+1)^2\pi^2}{4}\eta\right)$$

$$\tau=\frac{a^2}{l^2}t,\quad \Sigma(\tau)=\frac{1}{E}\sigma\left(\frac{l^2}{a^2}\tau\right),\quad \omega=\frac{fv\alpha El}{\lambda},\quad \chi_0=\alpha T_0$$

The last four nondimensional quantities are connected to the Péclet and Fourier criteria by simple relationships.

The Volterra integral equation (3) will be an equation of convolution type, and its solution $\Sigma(\tau)$ can be obtained by the customary method, by using a Fourier transform, say [9]. Let us represent the solution in two forms

$$\Sigma(\tau)=\frac{1}{\sqrt{2\pi}}\int_{ic-\infty}^{ic+\infty}\frac{Q(w)}{1-\sqrt{2\pi}K(w)}\exp(-i\tau w)\,dw \qquad (4)$$

$$\Sigma(\tau)=q(\tau)+\int_0^{\tau}q(s)\,m(\tau-s)\,ds \qquad (5)$$

$$m(\mu)=\frac{1}{\sqrt{2\pi}}\int_{ic-\infty}^{ic+\infty}M(w)\exp(-i\mu w)\,dw,\qquad M(w)=\frac{K(w)}{1-\sqrt{2\pi}K(w)}$$

The functions denoted by the same upper and lower case latin letters will be reciprocal Fourier transforms. Utilizing the expression for $k(\zeta)$ we obtain

$$M(w)=-\frac{1}{\sqrt{2\pi}}\left[\frac{1}{\omega}\,\frac{iw}{1-\operatorname{sch}\sqrt{-iw}}+1\right]^{-1}$$

The branch $\sqrt{-iw}$ is selected so that $\arg w=0$ will correspond to $\arg\sqrt{-iw}=$ $=-{}^{1}/_{4}\pi$. The function $M(w)$ will be meromorphic in the whole plane, with poles on the imaginary axis. As $\omega\to\infty$ the zeros of the denominator of the function $M(w)$ are disposed at the points

$$w_m=-4\pi^2m^2(1\pm2\sqrt{2}\,\omega^{-1})\,i,\qquad w^+=iJ^+\cdot\qquad(m=1,2,\ldots)$$

For J^+ the following estimate holds

$$J^+=\omega(1+\rho\operatorname{sch}\sqrt{\omega}),\qquad 0<\rho<{}^{1}/_{2},\qquad \omega\to\infty$$

For $\omega\to0$ the zeros are located at the points

$$w_m{}^{\bullet}=-\left[\frac{\pi^2}{4}(2m+1)^2+\frac{4}{\pi}\,\frac{(-1)^{m+1}}{2m+1}\,\omega\right]i$$

Two points $w_m{}^{\bullet}$ exist between each two adjacent points w_m; as ω varies between ∞ and 0, the zeros move from each point w_m (which will be first order branch points for $\omega = \infty$) to the adjacent two points $w_m{}^\circ$. For $\omega = 0$ and $\omega = 2$ the zero w^+ will be at the points $-{}^1/_4\pi^2 i$ and 0 , respectively.

Table 1.

ω	$T(x,0)=T_0,\ \Sigma(0)=0$		$T(x,0)=0,\ \Sigma(0)=\Sigma_0$	
	$\Sigma(t)$	$T(0,\tau)$	$\Sigma(\tau)$	$T(0,\tau)$
$\to 0$	$\Sigma(t)$ *	$T(0,\tau)$ **	$(1+{}^1/_2\omega)\,\Sigma_0$	${}^1/_3\,\omega^2\alpha^{-1}\Sigma_0$
2	${}^4/_5\chi_0$	${}^8/_5 T_0$	${}^{12}/_5\,\Sigma_0\tau$	${}^{24}/_5\alpha^{-1}\Sigma_0\tau$
$\to\infty$	$\chi_0 e^{\omega\tau}$	$2T_0 e^{\omega\tau}$	$\Sigma_0 e^{\omega\tau}$	$2\alpha^{-1}\Sigma_0 e^{\omega\tau}$

$$* \ \Sigma(\tau) = 2\frac{\chi_0}{\omega}\sin\frac{4\omega}{\pi^2}\exp\left[-\left(\frac{\pi^2}{4}-\frac{4}{\pi}\,\omega\right)\tau\right]$$

$$** \ T(0,\tau) = \frac{16}{\pi^2}\,T_0\omega^2\tau^2\exp\left(-\frac{\pi^2}{4}\tau\right)$$

The asymptotic representation of $\Sigma(\tau),\ \tau\to\infty$ is determined by the residue at the point w^+; the actual determination of $\Sigma(\tau)$ is performed by using the relationships (4), (5) (results of computations for values $0.2, \infty$ of the parameter ω are presented in Table 1; for intermediate values of ω the integral in the right side of (5) can be estimated by the method in [10]). We find the asymptotic representation for $\omega\to 2,\ \tau\to\infty$ from (4)

$$\Sigma(\tau) \sim {}^4/_5\,\chi_0\exp\left[{}^8/_5\,(\omega-2)\,\tau\right]$$

It hence follows that the stability of the considered friction process is determined by the condition $\omega \leqslant 2$, instability occurs for $\omega > 2$; the critical value of the velocity corresponding to the passage from the stable to the unstable friction mode is determined by the relationship

$$v_* = \frac{2\lambda}{f\alpha El} \tag{6}$$

In the second case, when the initial temperature of the plate is zero, and there are initial stresses, the integral equation (3) becomes

$$\Sigma(\tau) = \int_0^\tau k(\tau-s)\,\Sigma(s)\,ds + d(\tau),\quad d(\tau) = \Sigma_0\int_0^\tau k(s)\,ds,\qquad \Sigma_0 = \Sigma(0)$$

and the solution of this equation can be conducted by the same means as in the first case. For $\omega\to 2,\ \tau\to\infty$ we have

$$\Sigma(\tau) \sim 2\Sigma_0\frac{1}{\omega-2}\left\{\exp\left[{}^8/_5\,(\omega-2)\,\tau\right]-1\right\}$$

Hence, for $\omega = 2$ we find

$$\Sigma(\tau) \sim {}^{16}/_5\,\Sigma_0\tau$$

Therefore, stability in this case is determined by the condition $\omega < 2$.

The temperature in the friction zone is determined by using (1), here not the asymptotic but the exact value of $\Sigma(\tau)$ corresponding to all poles of $M(\omega)$ should be used.

We present the results of the calculations for $\tau \to \infty$ in Table 1.

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ON THE STEADY MOTIONS OF A GYROSTAT SATELLITE

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Two families of steady motions of a gyrostat satellite in a central Newtonian force field are considered. The plane of the (circular) orbit of the center of mass of the satellite is biased relative to the attracting center. Sufficient conditions for stability are derived.

These motions complement the numerous already familiar [1] steady motions of a gyrostat satellite with the center of the circular orbit coincident with the attracting center. As in the case of the latter motions, the stability conditions in our case differ from those obtained under the restricted formulation of the problem [1] by quantities on the order of l^2 / R^2 relative to the principal terms (l is the characteristic dimension of the satellite, R is the distance from the attracting center). The orbital plane bias is of the order of l^2 / R. These quantities are very small indeed when one is dealing with real artificial earth satellites.

The present study is carried out by the Routh method with the aid of some results obtained by Rumiantsev [1].